Linear Equations and Functions

I. Basic Definitions and Theorems

A. Intercepts

1. If a line intersects the x-axis at the point $(a, 0)$, then the number a is called the <u>x-intercept</u>.
2. If a line intersects the y-axis at the point $(0, b)$, then the number b is called the <u>y-intercept</u>.

B. Slope

1. The <u>slope m</u> of the line through the points (x_1, y_1) and (x_2, y_2) is given by the formula

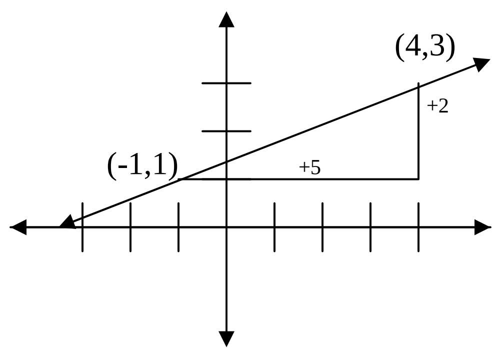
$$m = \frac{y_2 - y_1}{x_2 - x_1}, \text{ where } x_1 \neq x_2.$$

If $x_1 = x_2$, then we say that the line has no slope or an undefined slope. We can also define the slope by the formula

$$m = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$ where Δy is the <u>*change in y*</u> and Δx is the <u>*change in x*</u>.

Δy is a directed change (going up is +, going down is -) and Δx is also a directed change (going to the right is +, going to the left is -).

Example: Consider the line determined by the points $(-1,1)$ and (4, 3).



$$m = \frac{1-3}{-1-4} = \frac{-2}{-5} = \frac{2}{5}$$

or

$$m = \frac{3-1}{4-(-1)} = \frac{2}{5}$$

2. If the line L_1 has slope m_1 and line L_2 has slope m_2, then

(a) $L_1 \parallel L_2$ iff $m_1 = m_2$, and

(b) $L_1 \perp L_2$ iff $m_1 = -\dfrac{1}{m_2}$ where $m_1 \neq 0$ and $m_2 \neq 0$.

II. Equations of a Line

A. Standard form: $Ax + By = C$ where A and B are not both equal to 0.

B. Vertical line: $x = k$ (the line has undefined slope)

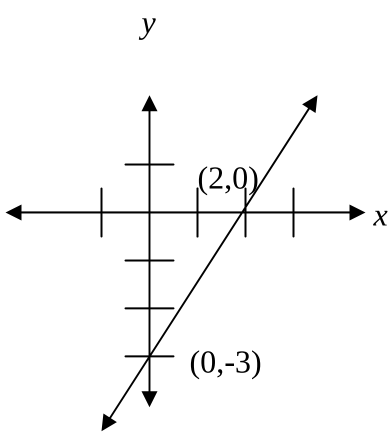
C. Horizontal line: $y = k$ (the line has slope 0)

D. Slope-intercept form: $y = mx + b$ where m is the slope and b is the y-intercept.

E. Point-slope form: $y - y_1 = m(x - x_1)$ or $\dfrac{y - y_1}{x - x_1} = m$ where m is the slope and the line passes through the point (x_1, y_1).

Example 1: Given a line with equation $3x - 2y = 6$, find the intercepts, the slope, and sketch the graph.

Solution 1: To find the x-intercept, let $y = 0 \Rightarrow 3x = 6 \Rightarrow x = 2$ is the x-intercept. To find the y-intercept, let $x = 0 \Rightarrow -2y = 6 \Rightarrow y = -3$ is the y-intercept.



$$m = \frac{0 - (-3)}{2 - 0} = \frac{3}{2}$$

Solution 2: Take the equation $3x - 2y = 6$ and solve for y to get equation in the slope-intercept form $y = \frac{3}{2}x - 3 \Rightarrow m = \frac{3}{2}$ and b = y-intercpt = -3. To get the x-intercept, let $y = 0 \Rightarrow x = 2$ is the x-intercept. Get the same graph as above using the intercepts.

Example 2: Find the equation of the line which is determined by the points $(-2, 4)$ and (3, 1).

Solution 1: Use the point-slope form. First, find the slope $m = \frac{4-1}{-2-3} = -\frac{3}{5}$.

Next, use either point; for example, use (3, 1). $y - 1 = -\frac{3}{5}(x-2)$ or

$\frac{y-1}{x-3} = -\frac{3}{5}$ or $3x + 5y = 14$ or $y = -\frac{3}{5}x + \frac{14}{5}$ are all correct equations.

Solution 2: Use the slope-intercept form. First, find the slope $m = -\frac{3}{5}$ as above. Next, use either point; for example use (-2, 4). $y = mx + b \Rightarrow$

$y = -\frac{3}{5}x + b \Rightarrow 4 = -\frac{3}{5}(-2) + b \Rightarrow b = \frac{14}{5}$ $y = -\frac{3}{5}x + \frac{14}{5}$ is the equation.

Example 3: Find an equation of the line through (3, 4) and parallel to the line $5x + 3y = -8$.

$5x + 3y = -8 \Rightarrow y = -\frac{5}{3}x - \frac{8}{3} \Rightarrow m = -\frac{5}{3} \Rightarrow$ a line parallel to this line must have the same slope $\Rightarrow$ $y - 4 = -\frac{5}{3}(x-3)$ is an equation of this line.

III. Linear Functions

A. A function f is a linear function if $f(x) = mx + b$, where m and b are real numbers and $m \neq 0$.

B. Application of linear functions

Example: Suppose that the height and weight of college females are linearly related. A college female who is 60” tall weighs 100 lb and another college female who is 65” tall weighs 112 lb. Find an equation which relates the height and weight.

Consider the two “points” (60”, 100 lb) and (65”, 112 lb), and find the equation of the line determined by these points. $m = \frac{112lb - 100lb}{65'' - 60''} = \frac{12lb}{5''}$

and use the “point” (60”, 100 lb) $\Rightarrow$ $W - 100 = \frac{12}{5}(H - 60)$ or

$12H - 5W = 220$ or $H = \frac{5}{12}W + \frac{55}{3}$ or $W = \frac{12}{5}H - 44$.

Practice Sheet – Linear Equations and Functions

I. Graph each of the following:

(1) $3x + 2y = 6$

(2) $y = -3x + 1$

(3) $y = -2$

(4) $x = 3$

(5) $\frac{x}{4} - \frac{y}{2} = 1$

(6) $-2x + 7y = -3$

(7) $y = \frac{3}{4}x - 2$

(8) $x + y = -4$

II. Write an equation in any form for each of the following lines:

(1) Through (-1, 2), m = -3

(2) Through (2, 5), $m = \frac{3}{4}$

(3) Through (-2, 4), m = 0

(4) Through (3, -2), undefined slope

(5) Through (1, 2) and (3, -1)

(6) x-intercept = 2, y-intercept = -3

(7) Through (2, -1) and parallel to $y = 3x + 4$

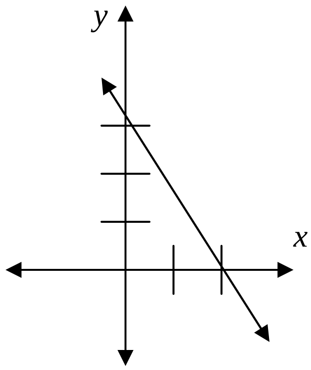
(8) Through (-3, -1) and perpendicular to $2x + 5y = 1$

III. (1) The height and weight of a college male are linearly related. If a college male who is 5'11" weighs 180 lbs and one who is 6'2" weighs 195 lbs, then how much should a college male who is 6'4" weigh?

(2) The price of a pair of basketball shoes is linearly related to the number purchased. If a shoe store purchases 100 pairs of shoes, then the price per pair is $90. However, if the store purchases 500 pairs of shoes, then the price per pair is $85. What would be the price per pair if the store purchases 750 pairs of shoes?

Solution Key for <u>Linear Equations and Functions</u>

I. (1)



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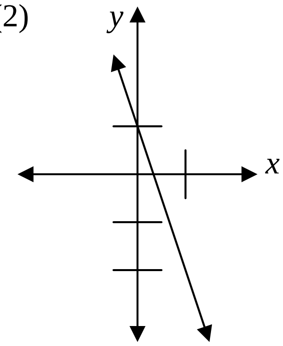

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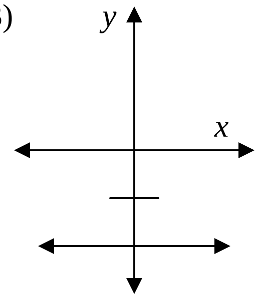

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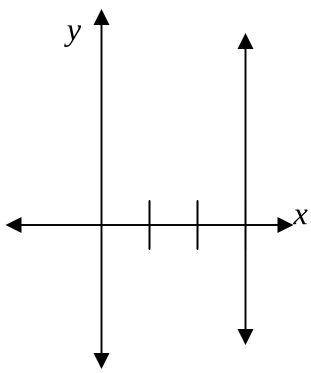

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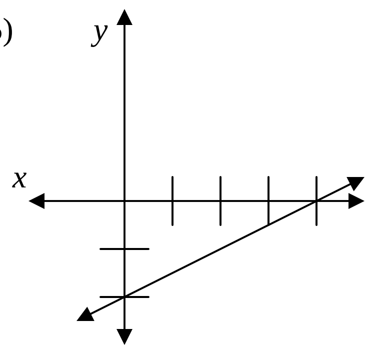

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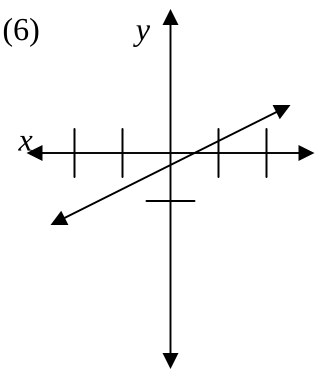

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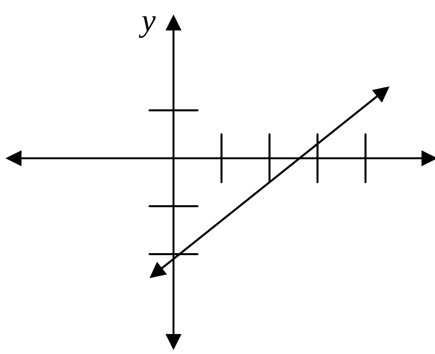

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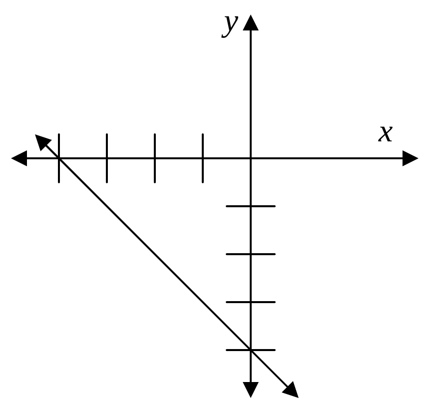

II. (1) $y = -3x - 1$

(2) $y = \frac{3}{4}x + \frac{7}{2}$

(3) $y = 4$

(4) $x = 3$

(5) $y = -\frac{3}{2}x + \frac{7}{2}$

(6) $y = \frac{3}{2}x - 3$

(7) $y = 3x - 7$

(8) $y = \frac{5}{2}x + \frac{13}{2}$ or $5x - 2y = -13$

III. (1) $W = 5H - 175 = 5(76) - 175 = 205$

(2) $C = -\frac{1}{80}S + 91.25 = -\frac{1}{80}(750) + 91.25 = \$81.88$